中2数学B　2019年度1学期　宿題解答
§4　ピタゴラス数

H4.1

(1)
$$\begin{aligned}(x+4)(x-16)-(x-8)(x+8)&=(x^2-12x-64)-(x^2-64)\\&=x^2-12x-64-x^2+64\\&=\boxed{-12x}\end{aligned}$$

(2)
$$\begin{aligned}2(x+3)(x-4)-(x-7)(x+5)&=2(x^2-x-12)-(x^2-2x-35)\\&=2x^2-2x-24-x^2+2x+35\\&=\boxed{x^2+11}\end{aligned}$$

(3)
$$\begin{aligned}\underbrace{\left(\sqrt{7}-\sqrt{2}\right)^2}_{\left(\sqrt{7}\right)^2-2\times\sqrt{7}\times\sqrt{2}+\left(\sqrt{2}\right)^2}-\underbrace{\left(2\sqrt{5}-\sqrt{11}\right)\left(2\sqrt{5}+\sqrt{11}\right)}_{\left(2\sqrt{5}\right)^2-\left(\sqrt{11}\right)^2}&=\left(7-2\sqrt{14}+2\right)-(20-11)\\&=9-2\sqrt{14}-9\\&=\boxed{-2\sqrt{14}}\end{aligned}$$

(4)
$$\begin{aligned}\left(6-2\sqrt{3}\right)\left(3\sqrt{2}+\sqrt{6}\right)-\underbrace{\left(3\sqrt{2}+2\right)^2}_{\left(3\sqrt{2}\right)^2+2\times3\sqrt{2}\times2+2^2}&=\left(18\sqrt{2}+\cancel{6\sqrt{6}}-\cancel{6\sqrt{6}}-2\times3\sqrt{2}\right)-\left(18+12\sqrt{2}+4\right)\\&=\left(18\sqrt{2}-6\sqrt{2}\right)-\left(22+12\sqrt{2}\right)\\&=12\sqrt{2}-22-12\sqrt{2}\\&=\boxed{-22}\end{aligned}$$

H4.2

(1) $$\frac{2}{\sqrt{10}-\sqrt{6}}-\frac{\sqrt{5}-\sqrt{3}}{\sqrt{2}}=\frac{2 \times\left(\sqrt{10}+\sqrt{6}\right)}{\underbrace{\left(\sqrt{10}-\sqrt{6}\right)\times\left(\sqrt{10}+\sqrt{6}\right)}_{10-6=4}}-\frac{\left(\sqrt{5}-\sqrt{3}\right)\times\sqrt{2}}{\sqrt{2}\times\sqrt{2}}$$

$$=\frac{\overset{1}{\cancel{2}}\left(\sqrt{10}+\sqrt{6}\right)}{\underset{2}{\cancel{4}}}-\frac{\sqrt{10}-\sqrt{6}}{2}$$

$$=\frac{\left(\sqrt{10}+\sqrt{6}\right)-\left(\sqrt{10}-\sqrt{6}\right)}{2}$$

$$=\frac{\sqrt{10}+\sqrt{6}-\sqrt{10}+\sqrt{6}}{2}$$

$$=\frac{2\sqrt{6}}{2}$$

$$=\boxed{\sqrt{6}}$$

(2) $$\frac{6}{\sqrt{7}-2}+\frac{4}{\sqrt{7}+3}=\frac{6\times\left(\sqrt{7}+2\right)}{\underbrace{\left(\sqrt{7}-2\right)\times\left(\sqrt{7}+2\right)}_{7-4=3}}+\frac{4\times\left(\sqrt{7}-3\right)}{\underbrace{\left(\sqrt{7}+3\right)\times\left(\sqrt{7}-3\right)}_{7-9=-2}}$$

$$=\frac{\overset{2}{\cancel{6}}\left(\sqrt{7}+2\right)}{\underset{1}{\cancel{3}}}+\frac{\overset{2}{\cancel{4}}\left(\sqrt{7}-3\right)}{-\underset{1}{\cancel{2}}}$$

$$=2\left(\sqrt{7}+2\right)-2\left(\sqrt{7}-3\right)$$

$$=2\sqrt{7}+4-2\sqrt{7}+6$$

$$=\boxed{10}$$

H4.3

$1<\sqrt{2}<2$ より、$\sqrt{2}$ の整数部分は1であり、ゆえに小数部分 a は

$$a=\sqrt{2}-1$$

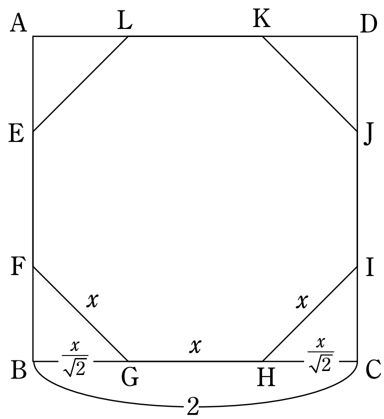
である。これより、

$$\frac{1}{a}=\frac{1}{\sqrt{2}-1}=\frac{1\times\left(\sqrt{2}+1\right)}{\left(\sqrt{2}-1\right)\left(\sqrt{2}+1\right)}=\frac{\sqrt{2}+1}{2-1}=\sqrt{2}+1$$

であるから、

$$\frac{1}{a}-a=\left(\sqrt{2}+1\right)-\left(\sqrt{2}-1\right)=\boxed{2}$$

H4.4



正八角形の1辺の長さを x とおくと、

直角二等辺三角形 BFG において、

$\mathrm{FG}:\mathrm{BG}=\sqrt{2}:1$ なので、$\mathrm{BG}=\dfrac{x}{\sqrt{2}}$

同様に、$\mathrm{CH}=\dfrac{x}{\sqrt{2}}$

したがって、

$$\begin{aligned}\mathrm{BC}&=\mathrm{BG}+\mathrm{GH}+\mathrm{HC}\\&=\frac{x}{\sqrt{2}}\times2+x=\sqrt{2}x+x=\left(\sqrt{2}+1\right)x\end{aligned}$$

正方形 ABCD の1辺の長さ BC は2だったので、

$$\left(\sqrt{2}+1\right)x=2$$

$$x=\frac{2}{\sqrt{2}+1}=\frac{2\left(\sqrt{2}-1\right)}{\underbrace{\left(\sqrt{2}+1\right)\left(\sqrt{2}-1\right)}_{\sqrt{2}^2-1^2=2-1=1}}=2\left(\sqrt{2}-1\right)$$

よって、正八角形の1辺の長さは、$\boxed{2\sqrt{2}-2}$