中2数学X　2019年度1学期　宿題解答
§6　平方差

H6.1

(1)　$x^2-144=x^2-12^2=\boxed{(x+12)(x-12)}$

(2)　$16x^2-49y^2=(4x)^2-(7y)^2=\boxed{(4x+7y)(4x-7y)}$

H6.2

(1)　$\left(\sqrt{7}-\sqrt{3}\right)\left(\sqrt{7}+\sqrt{3}\right)=\left(\sqrt{7}\right)^2-\left(\sqrt{3}\right)^2=7-3=\boxed{4}$

(2)　$\left(2\sqrt{3}-\sqrt{5}\right)\left(2\sqrt{3}+\sqrt{5}\right)=\left(2\sqrt{3}\right)^2-\left(\sqrt{5}\right)^2=4\times3-5=\boxed{7}$

H6.3

(1)　$$\frac{4}{\sqrt{7}+\sqrt{3}}=\frac{4\qquad\times\left(\sqrt{7}-\sqrt{3}\right)}{\underbrace{\left(\sqrt{7}+\sqrt{3}\right)\times\left(\sqrt{7}-\sqrt{3}\right)}_{\left(\sqrt{7}\right)^2-\left(\sqrt{3}\right)^2=7-3=4}}=\frac{\overset{1}{\cancel{4}}\left(\sqrt{7}-\sqrt{3}\right)}{\underset{1}{\cancel{4}}}=\boxed{\sqrt{7}-\sqrt{3}}$$

(2)　$$\frac{7\sqrt{2}}{2\sqrt{2}-1}=\frac{7\sqrt{2}\qquad\times\left(2\sqrt{2}+1\right)}{\underbrace{\left(2\sqrt{2}-1\right)\times\left(2\sqrt{2}+1\right)}_{\left(2\sqrt{2}\right)^2-1^2=4\times2-1=7}}=\frac{\overset{1}{\cancel{7}}\sqrt{2}\left(2\sqrt{2}+1\right)}{\underset{1}{\cancel{7}}}=2\times2+\sqrt{2}=\boxed{4+\sqrt{2}}$$

(3)

$$\frac{\sqrt{6}}{3-\sqrt{3}}=\frac{\sqrt{6}\quad\times\left(3+\sqrt{3}\right)}{\underbrace{\left(3-\sqrt{3}\right)\times\left(3+\sqrt{3}\right)}_{3^2-\left(\sqrt{3}\right)^2=9-3=6}}=\frac{\sqrt{6}\left(3+\sqrt{3}\right)}{6}=\frac{3\sqrt{6}+3\sqrt{2}}{6}=\frac{\overset{1}{\cancel{3}}\left(\sqrt{6}+\sqrt{2}\right)}{\underset{2}{\cancel{6}}}=\frac{\sqrt{6}+\sqrt{2}}{2}$$

$$\frac{2}{\sqrt{6}+\sqrt{2}}=\frac{2\quad\times\left(\sqrt{6}-\sqrt{2}\right)}{\underbrace{\left(\sqrt{6}+\sqrt{2}\right)\times\left(\sqrt{6}-\sqrt{2}\right)}_{\left(\sqrt{6}\right)^2-\left(\sqrt{2}\right)^2=6-2=4}}=\frac{\overset{1}{\cancel{2}}\left(\sqrt{6}-\sqrt{2}\right)}{\underset{2}{\cancel{4}}}=\frac{\sqrt{6}-\sqrt{2}}{2}$$

$$\therefore\frac{\sqrt{6}}{3-\sqrt{3}}-\frac{2}{\sqrt{6}+\sqrt{2}}=\frac{\sqrt{6}+\sqrt{2}}{2}-\frac{\sqrt{6}-\sqrt{2}}{2}=\frac{\sqrt{6}+\sqrt{2}-\left(\sqrt{6}-\sqrt{2}\right)}{2}$$

$$=\frac{\sqrt{6}+\sqrt{2}-\sqrt{6}+\sqrt{2}}{2}=\frac{2\sqrt{2}}{2}=\boxed{\sqrt{2}}$$

(4)

$$\frac{1}{5+\sqrt{22}}=\frac{1\quad\times\left(5-\sqrt{22}\right)}{\underbrace{\left(5+\sqrt{22}\right)\times\left(5-\sqrt{22}\right)}_{5^2-\left(\sqrt{22}\right)^2=25-22=3}}=\frac{5-\sqrt{22}}{3}$$

$$\frac{1}{\sqrt{22}+\sqrt{19}}=\frac{1\quad\times\left(\sqrt{22}-\sqrt{19}\right)}{\underbrace{\left(\sqrt{22}+\sqrt{19}\right)\times\left(\sqrt{22}-\sqrt{19}\right)}_{\left(\sqrt{22}\right)^2-\left(\sqrt{19}\right)^2=22-19=3}}=\frac{\sqrt{22}-\sqrt{19}}{3}$$

$$\frac{1}{\sqrt{19}+4}=\frac{1\quad\times\left(\sqrt{19}-4\right)}{\underbrace{\left(\sqrt{19}+4\right)\times\left(\sqrt{19}-4\right)}_{(\sqrt{19})^2-4^2=19-16=3}}=\frac{\sqrt{19}-4}{3}$$

$$\therefore\frac{1}{5+\sqrt{22}}+\frac{1}{\sqrt{22}+\sqrt{19}}+\frac{1}{\sqrt{19}+4}=\frac{5-\cancel{\sqrt{22}}}{3}+\frac{\cancel{\sqrt{22}}-\cancel{\sqrt{19}}}{3}+\frac{\cancel{\sqrt{19}}-4}{3}$$

$$=\frac{5-4}{3}=\boxed{\frac{1}{3}}$$

H6.4

(1) $\dfrac{120^2-97^2}{91^2-70^2}=\dfrac{(120+97)(120-97)}{(91+70)(91-70)}=\dfrac{\overset{31}{\cancel{217}}\times\cancel{23}}{\underset{7}{\cancel{161}}\times 21}=\boxed{\dfrac{31}{21}}$

(2) $n^2-(n+1)^2=\{n+(n+1)\}\{n-(n+1)\}=-\{n+(n+1)\}$ であるから、

$$\begin{aligned}S&=(1^2-2^2)+(3^2-4^2)+\cdots+(99^2-100^2)\\&=-(1+2)-(3+4)-\cdots-(99+100)\\&=-(1+2+3+4+\cdots+99+100)\\&=-(1+100)\times100\div2=\boxed{-5050}\end{aligned}$$

H6.5

$\sqrt{n^2+92}$ が整数であるとし、 それを m (もちろん正)とおくと、

$n^2+92=m^2$

$92=m^2-n^2$

$\therefore (m+n)(m-n)=92$ ………………………… ①

ここで、整数 $m+n,\ m-n$ について、以下が成り立つことに注意する。

- ①より $m+n$ と $m-n$ は同符号であり、 m,n は正だから $m+n>0,\ m+n>m-n$
  よって、 $m+n>m-n>0$
- $(m+n)+(m-n)=2m$ は偶数だから、 $m+n,\ m-n$ の偶奇は一致

したがって、①より、

$$\begin{cases}m+n\\m-n\end{cases}=\cancel{\begin{cases}92\\1\end{cases}},\ \begin{cases}46\\2\end{cases},\ \cancel{\begin{cases}23\\4\end{cases}}$$

であるから、

$(m,n)=(24,22)$

以上より、求める n は $\boxed{n=22}$ である。