中 3 数学 D　復習テスト解答（3 学期-6）

復習 6-1

(1) $\displaystyle\sum_{k=1}^{5} k = 1+2+3+4+5 = \boxed{15}$

(2) $\displaystyle\sum_{k=1}^{5} 2^k = 2^1+2^2+2^3+2^4+2^5 = \boxed{62}$

(3) $\displaystyle\sum_{k=1}^{5} (k+2^k) = \sum_{k=1}^{5} k + \sum_{k=1}^{5} 2^k = 15+62 = \boxed{77}$

(4) $\displaystyle\sum_{k=1}^{5} \frac{k}{2^k} = \frac{1}{2^1}+\frac{2}{2^2}+\frac{3}{2^3}+\frac{4}{2^4}+\frac{5}{2^5} = \boxed{\frac{57}{32}}$

復習 6-2

(1) $\displaystyle\sum_{k=1}^{n} (2k+1) = 2\sum_{k=1}^{n} k + \sum_{k=1}^{n} 1 = 2\cdot\frac{n(n+1)}{2} + n$

$\displaystyle = n^2+2n = \boxed{n(n+2)}$

(2) $1\cdot3+2\cdot5+3\cdot7+\cdots+(n-1)\cdot(2n-1)$

$\displaystyle = \sum_{k=1}^{n-1} k(2k+1) = \sum_{k=1}^{n-1} (2k^2+k) = 2\sum_{k=1}^{n-1} k^2 + \sum_{k=1}^{n-1} k$

$\displaystyle = 2\cdot\frac{1}{6}(n-1)n(2n-1) + \frac{1}{2}(n-1)n$

$\displaystyle = \frac{1}{6}(n-1)n\cdot\{2(2n-1)+3\}$

$\displaystyle = \boxed{\frac{1}{6}(n-1)n(4n+1)}$

(3) m を正の整数とするとき，

$k = m^2, m^2+1, \cdots, \underbrace{(m+1)^2-1}_{m^2+2m}$

の $2m+1$ 個の k に対して，$[\sqrt{k}] = m$ となるので，

$[\sqrt{1}], [\sqrt{2}], \cdots, [\sqrt{n^2}]$

は, 1 が 3 個, 2 が 5 個, ……, $n-1$ が $2n-1$ 個，n が 1 個並んだ数列である.

したがって，(2)の結果を利用して，

$\displaystyle\sum_{k=1}^{n^2} [\sqrt{k}] = \left\{\sum_{k=1}^{n-1} k(2k+1)\right\} + n$

$\displaystyle = \frac{1}{6}(n-1)n(4n+1)+n$

$\displaystyle = \frac{1}{6}n\{(n-1)(4n+1)+6\}$

$\displaystyle = \boxed{\frac{1}{6}n(4n^2-3n+5)}$